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ing the other *BD* in *K*. But it will meet it in some point *K* existing between the points *B* and *D;* otherwise (contrary to Eu. I. 17) the perpendicular *LK* would cut either *AB*, or *CD*, perpendicular to the same *BD*. So if the angles at the point *L* are not right, one of them will be acute and the other obtuse. Let the obtuse be toward the point *C*. But now *LK* is understood so to proceed toward *AB*, that it always stands at right angles to *BD*, and likewise opportunely increased, or diminished, in some point of it cuts the straight *AC*. It follows that the angles at the intersection points with *AC* cannot all be obtuse toward the parts of the point *C*, lest at length in that point *A*, where the straight *LK* is congruent with the straight *AB*, the angle at the point *A* toward the parts of the point *C* should be obtuse, when toward these parts it is by hypothesis acute. Since therefore the angle at the point *L* of this *LK* is by hypothesis obtuse toward the parts of the point *C*, the straight *LK* will not change over in this motion so as to make in some point of it with the straight *AC* an angle acute toward the parts of the aforesaid point *C*, unless, before, it changes over so as to make in some point of it with this *AC* an angle right towards the parts of this same point *C*. Therefore between the points *A*, and *L* will be some one intermediate point *H*, in which *HK* perpendicular to this *BD* is also perpendicular to to the other *AC*.

In a similar manner is shown to be present a certain *XK* between *LK*, *CD*, which is perpendicular both to the straight *BD*, and to the straight *AC*, if namely an angle at the point *L* is assumed to be obtuse toward the parts of the point A.

It follows therefore that the strights *AC*, *BD* will have a common perpendicular, and indeed within the limits fixed by the designated points *A*, and *C*, when the joins *AB*, *CD* exist in the same plane and are perpendicular to *BD*.

Quod erat, etc.

[To be Continued.]

INTRODUCTION TO SUBSTITUTION GROUPS.

By G. A. MILLER, Ph. D., Leipzig, Germany.

[Continued from January Number.]

(2) For each generating substitution s_α in a transitive group of degree m_1 find a substitution s_β which (*a*) interchanges the systems in the same way as s_α interchanges its elements, (*b*) has its k^{th} power in G_1 where k is the *order* of s_α, *i. e.* the lowest positive value of x which satisfies the equation $s_\alpha^x=1$, and (*c*) if s_β is

the first substitution which corresponds to a generating substitution in the group of degree m_1, s_β needs only to transform G_1 into itself; otherwise s_β must transform the group already found in the same way as s_α transforms the corresponding part of its group. Continue until all the generating substitutions s_α have been used. We will thus obtain a non-primitive group.

(3) Determine whether the non-primitive group just found is different from each one of those already in the list.

The relation which exists between the required non-primitive group G and the given group of degree m_1 G^1 is called a g_1, 1 *isomorphism*, or a g_1, 1 *correspondence*. The problem of constructing all the non-primitive groups of degree n has its more difficult elements in common with the problem of establishing an a, b correspondence between two groups as may at once be inferred from the given relation. We shall not pursue this subject for the present since only its most evident principles need to be employed when n is small.

To this development of the elementary methods pursued in the construction of non-primitive groups we will add a proof of the general theorem to which we referred in a foot-note. For the sake of simplicity we shall not give the theorem in its most general form.

Theorem. Given that the number of the systems of non-primitivity is n and that the group which does not interchange the systems G_1 is the product of n conjugate transitive groups of which one is found in each system, then there is only one non-primitive group based upon the given G_1 and isomorphic to a transitive group of n elements which is generated by a single substitution.

There certainly is one such group for we may chose s_β so that it will simply permute the systems in the same way as s_α permutes its elements and will have the same order as s_α. Since s_β simply permutes the systems, *i. e.* it permutes the systems as units without permuting the elements of the systems, it must also transform G_1 into itself. · Hence G_1 and s_β generate a non-primitive group whenever G_1 differs from identity.

Let t_1^1, t_2^2, ... t_n^n(the upper index standing for the systems in the same order as they are represented by the letters of s_α and the lower index for the particular substitution in the system) be any substitution in the n systems which transform the n constituents of G_1 into themselves. Then will

$$t_1^1\, t_2^2 \,\ldots\ldots t_n^n s_\beta$$

be a symbol for all the substitutions whose degree $\leqq$ the degree of the required group which transform G_1 into itself and permute the systems in the same way as s_α permutes its elements. If this general substitution satisfies the other condition which must be satisfied if it, with the given G_1, generates a non-primitive group we have

$$(t_1^1\, t_2^2 \,\ldots\ldots t_n^n s_\beta\,)^K = \text{some substitution in } G_1,$$

where K is the smallest positive value of x in the equation

$$s_a^x=1.$$

Since $(t_1^1\, t_2^1 \ldots\ldots t_n^n s_\beta)^K = t_1^1\, t_2^1 \ldots\ldots t_n^1\, t_2^2\, t_3^2 \ldots\ldots t_1^2 \ldots\ldots t_n^n t_1^n \ldots\ldots t_{n-1}^n$

we know that $t_1^1\, t_2^2 \ldots\ldots t_n^n s_\beta$ may be multiplied by some substitution of G_1 so as to give for the new t's

$$t_1^1\, t_2^2 \ldots\ldots t_n^n=1 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (A)$$

Consider now the equations

$$(K_1^1\, K_2^2 \ldots\ldots K_n^n)^{-1} s_\beta\, K_1^1\, K_2^2 \ldots\ldots K_n^n{}^{*}=$$

$$\overset{-1}{K}{}_n^n K_1^n\, \overset{-1}{K}{}_{n-1}^{n-1} K_n^{n-1} \ldots\ldots \overset{-1}{K}{}_1^1\, K_2^1\, s_\beta = t_1^1\, t_2^2 \ldots\ldots t_n^n s_\beta\, .$$

We see directly that the following is a solution of the last equation if (A) is satisfied :

$$K_1^1=1,\ K_2^2=t_1^2\,,\ K_3^3=t_1^3\, t_2^3\,, \ldots\ldots K_n^n=t_1^n\, t_2^n \ldots\ldots t_{n-1}^n.$$

Hence all the possible groups are conjugate to the one already given and our theorem is proved. This theorem may be employed with respect to the first subgroups as well as with respect to the entire groups.

In our next paper we shall consider the construction of the third and last class of groups, viz: the *primitive* groups.

[To be Continued.]

ON AN INTERESTING SYSTEM OF QUADRATIC EQUATIONS.

By **DR. E. H. MOORE**, University of Chicago, and **EMMA C. ACKERMANN**, Michigan State Normal School.

In C. Smith's Algebra, fourth edition, p. 134, are given for solution, examples 61, 62, 63, which are as follows (the third with a slight modification):

61. The roots of the equation $x^2+mx+m^2+a=0$ are x_1, x_2 ; show that $x_1{}^2+x_1x_2+x_2{}^2+a=0$.

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*This equation follows from the simpler one

$$(ts)^{-1}=s^{-1}t^{-1}$$

and this is true because if we multiply both members by ts we obtain an identity.